Astroparticle Physics, NAASPH-12.2015-2016.2A

Ad van den Berg & Olaf Scholten
KVI - Center for Advanced Radiation Technology NL-9747 AA Groningen

Exam; March 30, 2016; 14:00-17:00; A. Jacobshal 02
5 problems (total of 50 points).
Write the solution of problems 1 and 2 and of problems 3, 4, and 5 on separate pieces of paper with name and student number.

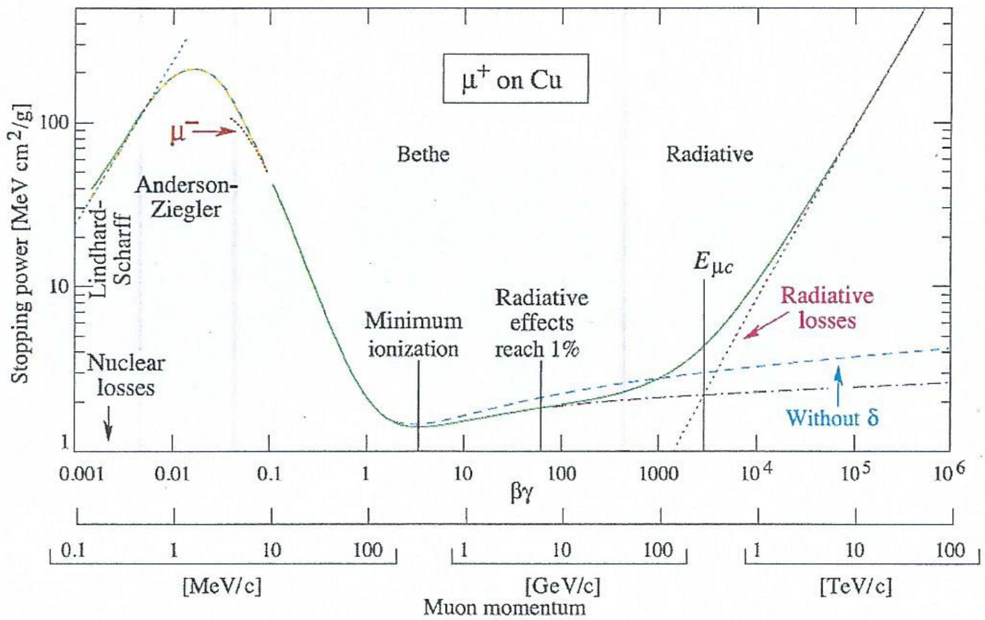
Write clearly, it should be readable.

Problem 1 (10 pnts in total)
In the graph below, you see the differential energy loss of charged particles, while they penetrate matter. In the present case muons in copper.

4 pnts a. Explain the mechanism for the energy loss between $\beta \times \gamma \approx 0.1$ and $\beta \times \gamma \approx 2000$ (until $E_{\mu c}$) labeled as "Bethe" in the figure.

4 pnts b. Explain the mechanism for the energy loss for $\beta \times \gamma \geq 2000$ (roughly $E_{\mu c}$) labeled as "Radiative" in the figure.

2 pnts c. What is the importance of the parameter E_c (critical energy) in shower physics.



Problem 2 (12 pnts in total)

A photon hits the top of the atmosphere with an initial energy E_0. This creates an electromagnetic shower in the atmosphere, which can be modeled following a theory developed by Heitler.

2 pnts a. Explain the two physics processes which rule the development of this electromagnetic shower.

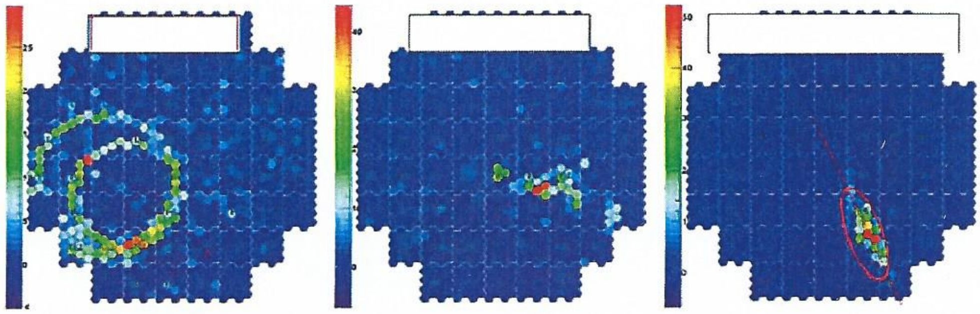
2 pnts b. Provide the expression in the Heitler model for the number of particles in the shower as a function of the penetration depth t into the atmosphere.

2 pnts c. Provide an expression in the Heitler model for the average energy of the individual particles as a function of the penetration depth t into the atmosphere.

2 pnts d. Provide an estimate of the maximum number of particles which is being created in the shower.

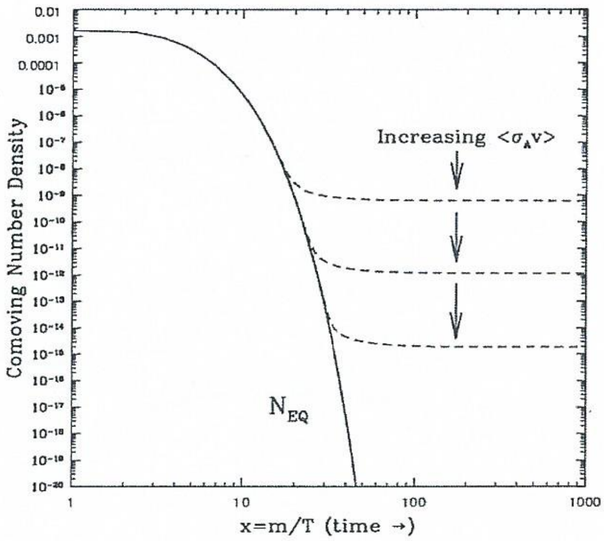
2 pnts e. In the picture below you see images (Left, Middle and Right panels) of three different showers as measured at the focal plane of an Imaging Air Cherenkov Telescope. The three different picture have been caused by: an electromagnetic shower, a hadronic shower, and high-energy muons; all traveling through the atmosphere. The color coding is a scale for the light intensity measured per unit of the surface on the focal plane. Correlate these three different pictures with the three different showers.

2 pnts f. Provide an explanation for every choice you make.



Problem 3 (9 pnts in total)
There are many indications that the energy density in the Universe today should be attributed for more than 20% to Dark Matter (DM). The figure shows the density of DM in the universe as function of time or temperature. The time-evolution of the density n_χ of of DM particles is determined by

$$\frac{dn_\chi}{dt} + 3H\, n_\chi = -\langle\sigma_\chi v\rangle \left[n_\chi^2 - (n_\chi^{eq})^2\right]$$



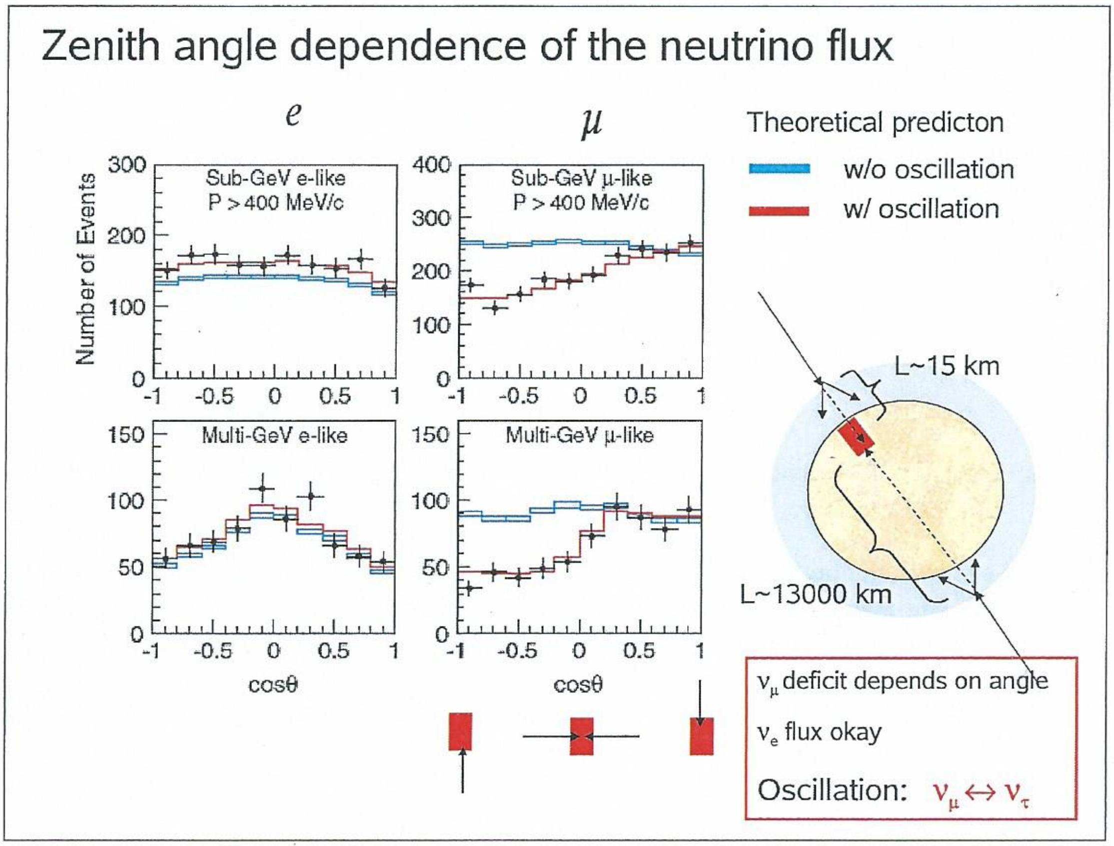
4 pnts a. Describe the physics for each of the four terms in the equation for the time evolution of the DM density.

2 pnts b. As shown in the figure the density of DM particles can be high during the early stage after the Big Bang, but becomes constant at a certain stage. Explain why this happens and why the final density decreases with increasing $<\sigma_\chi v>$ as indicated.

3 pnts c. Describe in no more than 2 or 3 sentences the principle of the DAMA experiment.

Problem 4 (10 pnts in total)

At the lecture the results of Kamiokande for (so called) 'atmospheric' neutrino oscillations were presented in this figure.



The black dots in the figures show the results of the Kamiokande experiment for the number of detected electron and muon neutrinos as function of the cosine of the angle ($\theta = 0°$ is from above; zenith, $\theta = 180°$ is from below; nadir) where the blue line is the expected count rate in the absence of neutrino oscillations. The red line is the expected count rate when realistic neutrino oscillations are taken into account.

2 pnts a. Argue that for $\cos\theta = 1$ you do not expect any sizable disappearance due to neutrino oscillations.

4 pnts b. Show that for electron neutrinos you expect only a minor disappearance of flux at $\cos\theta = -1$ for 1 GeV neutrinos.

2 pnts c. Argue that the effect of neutrino oscillations for electron neutrinos is smaller when their energy is larger.

2 pnts d. Show that for 1 GeV muon neutrinos the effect of oscillations is close to maximal at $\cos\theta = -1$.

Problem 5 (9 pnts in total)
A high energy electron in a shower scatters off a nucleon (mass M, at rest) in an air-molecule. Assume that the electron, moving in the $\hat{z}$-direction, has an energy of $E_e = 10^{16}$ eV and scatters over a small angle θ of order mili-radians. In the scattering process it has lost half its energy and thus $E'_e = 5 \times 10^{15}$ eV. The electron has transferred energy and momentum (four momentum q^μ) to one of the quarks inside the nucleon which has a momentum fraction x_{bj}.

3 pnts a. Calculate the components of q^μ. Show that for the energies and angles in this problem $q^2 \approx -2\,E\,E'(1-\cos\theta) \approx -10^{32}\theta^2/2$ eV^2 for $m_e \ll E$.

1 pnts b. Show that the momentum fraction of the parton is given by $x_{bj} = -q^2/(2M(E-E'))$.

2 pnts c. Indicate in a simple figure where a color-string will be formed.

2 pnts d. Calculate the invariant mass of the color-string that is formed when the electron scattering angle equals $\theta = 10^{-4}$ radian.

Some numbers

Electron mass $m_e c^2 = 511\,$keV; Muon mass $m_\mu c^2 = 106\,$MeV; Pion mass $m_\pi c^2 = 140\,$MeV;
Proton mass: $m_p c^2 = 0.938\,$GeV
Conversion: $1\,\text{eV}/c^2 = 1.78 \times 10^{-36}\,$kg
Boltzmann's constant: $k = 8.62 \times 10^{-11}\,$MeV/K
Planck's constant: $h = 4.1 \times 10^{-15}\,$eV s
Avogadro's number: $N_A = 6 \times 10^{23}\,$/mol
Solar Mass: $M_\odot = 1.99 \times 10^{30}\,$kg
Parsec: $1\,\text{pc} = 3.1 \times 10^{16}\,$m
Velocity of Sun w.r.t. center Milkyway: $V_\odot = 270\,$km/s
Velocity of Earth in orbit around the Sun: $V_\oplus = 30\,$km/s
Radius of Earth: $R_\oplus = 6.4 \times 10^3$ km
Typical galactic dark matter density: $\rho_{DM} = 9\ \text{k}\ M_\odot/\text{pc}^3$

Neutrino-oscillation physics

$\Delta m^2_{atm} = 2.5 \times 10^{-3}$ eV^2; $\Delta m^2_{sol} = 0.8 \times 10^{-4}$ eV^2
$P(\nu_\alpha \to \nu_\alpha) = 1 - \sin^2 2\theta\, \sin^2 \frac{1.27\ \Delta m^2 L}{4E}$
with Δm^2 in [eV^2]; L in [km]; E in [GeV]; and θ the neutrino mixing angle.

Air-shower physics

At 10 km height the density of the atmosphere is $0.4 \times 10^{-3}\,\text{g cm}^{-3}$.
The penetration depth for pions in air is $\lambda_\pi = 120\,\text{g cm}^{-2}$, for protons $\lambda_p = 90\,\text{g cm}^{-2}$, and for iron is $\lambda_{Fe} = 5\,\text{g cm}^{-2}$.
The mean travel distance in vacuum of a pion with energy E is $d_{\pi^0} = \gamma 25 \times 10^{-9}\,$m and $d_{\pi^\pm} = \gamma 7.8\,$m where the relativistic γ-factor is given by $\gamma = E/mc^2$ and $m_\pi c^2 = 140\,$MeV.

Integrals

For $c > 0$ we have:

$$\int_0^\infty c\,e^{-cx} dx = 1\,; \quad \int_0^\infty c\,x\,e^{-cx} dx = 1/c\,; \quad \int_0^\infty c\,x^2 e^{-cx} dx = 2/c^2$$